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Foreign Technology Division
Wright-Patterson Air Force Base, Ohio

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by

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An optimal statistical treatment algorithm, valid for wide and narrow band processes, is obtained for the detection of a weak harmonic signal on a gaussian narrow band noise background. The integral is based on the statistics of intervals between intersections of zero level processes. It is shown that the noise rejection capacity of a detection process increases when the absolute signal mismatch level with respect to the central frequency of the noise spectrum increases.

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III

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

WEAK HARMONIC-SIGNAL DETECTION IN GAUSSIAN NARROW-BAND NOISE USING THE STATISTICS OF INTERVALS BETWEEN ZEROES

I. N. Yerimichoy and V. M. Koshevoy

Engineers

Recently there have appeared many works devoted to receivers in which a rigid limitation at the input is used. Systems for detecting a harmonic signal, which use optimum phase treatment of the input process [6], the numbers of passages through "zero" in a given time interval [1, 5], and moments of passage through "zero," were examined.

Let us examine the question of the optimum detection of a weak harmonic signal in narrow-band gaussian noise using the statistics of intervals between "zeroes." The problem is solved based on the theory of checking statistical hypotheses.

Beginning with the proposition of the nondependence of the access elements $y = (y_1, y_2, \ldots, y_n)$, let us write the probability ratio

$$l(y) = \frac{\prod_{i=1}^{n} w_1(y_i/a)}{\prod_{i=1}^{n} w_1(y_i)}.$$

Let us determine the density of the probability of intervals between zeroes for the case of the presence of the useful signal. Let $\xi(t)$ be the normal random process which is the sum of harmonic signal $A_m \cos \omega_c t$ and narrow-band noise $n(t)$ with a zero average and dispersion

$$\xi(t) = A_m \cos \omega_c t + n(t) = E(t) \cos [\omega_c t + \varphi(t)]. \tag{1}$$

The interval between adjacent zeroes involves a derivative from the phase of the random process by the following expression [7]:

$$\tau = \frac{\pm \pi}{\Phi'}, \tag{2}$$

where

$$\Phi' = \omega_c + \varphi'.$$

Since the probability of negative values Φ' is very slight $P(\Phi' < 0) \simeq 0$ [2], then

$$\tau = \frac{\pi}{\Phi'} = \frac{\pi}{\omega_c + \varphi'}. \tag{3}$$

Instead of τ it is more convenient to examine the random magnitude $y = \tau \frac{\omega_0}{\pi}$, where ω_0 is the central frequency of the noise power spectrum.

$$y = \frac{\omega_0}{\pi} \frac{\pi}{\omega_c + \varphi'} = \frac{1}{1 - q\sqrt{k} + x\sqrt{k}}, \tag{3'}$$

where $x = \frac{\varphi'}{\omega_*}$; $\omega_* = \frac{1}{\sigma}\left[\int_0^\infty (\omega - \omega_0) F(\omega)\, d\omega\right]^{\frac{1}{2}}$ – is the average square width of the noise spectrum, and $k = \frac{\omega_*^2}{\omega_0^2}$; $q = \frac{\omega_0 - \omega_c}{\omega_*}$ – is the standardized detuning of the signal frequency from the central frequency of the noise spectrum.

Using equation (3') we can express the density of the probability of a random value of y in terms of the density of

random value x

$$W_1(y) = \omega_1(x)\left|\frac{dx}{dy}\right|. \qquad (4)$$

The expression of the probability closeness $\omega_1(x)$ for random process (1) is shown in the equation [4]:

$$\omega_1(x/a) = \omega_0(x)\left[(1+a\nu) I_0\left(\frac{a(1-\nu)}{2}\right) + \right.$$

$$\left. + a\gamma I_1\left(\frac{a(1-\nu)}{2}\right)\right] e^{-\frac{a(1-\nu)}{2}}. \qquad (5)$$

where $\omega_0(x) = \frac{1}{2}[1+(q-x)^2]^{-\frac{3}{2}}$; $\nu = \frac{x^2}{1+(q-x)^2}$; $\gamma = \frac{(1+q^2-qx)^2}{1+(q-x)^2}$; $a = \frac{A_m^2}{2\sigma^2}$ is the ratio of signal power and noise power.

For low signal-to-noise ratios ($a \ll 1$), using the expansion of the Bessel and exponential functions, equation (5) can be rewritten as follows:

$$\omega_1(x/a) \simeq \omega_0(x)\left[1 - a\left(\frac{1}{2} + \frac{1}{2}\nu - \gamma\right)\right]. \qquad (6)$$

Using formulas (3'), (4), and (6), we can write the probability density of the random magnitude y:

$$W_1(y/a) = W_1(y)\left\{1 - a\left[1 - q^2 - \left(\frac{3}{2} + \frac{3q}{\sqrt{k}} - \frac{3}{2}q^2\right)v^{-1} + \frac{3q}{\sqrt{k}}(yv)^{-1}\right]\right\}. \qquad (7)$$

where

$$W_1(y) = \frac{1}{2\sqrt{k y^2}} v^{-\frac{3}{2}}. \qquad (8)$$

The probability density of standardized intervals between zeroes in the absence of useful signal

$$v = 1 + \frac{1}{k}\left(1 - \frac{1}{y}\right)^2. \qquad (9)$$

The obtained probability density coincides with Byaliy's results [2], while for low signal-to-noise ratios it coincides with the result obtained in work [7].

Using equations (7) and (8), let us write the logarithm of the probability ratio

$$\ln l(y) = \sum_{i=1}^{n} \ln \left\{ 1 - a\left[1 - q^2 - \left(\frac{3}{2} + \frac{3q}{\sqrt{k}} - \frac{3}{2} q^2 \right) v^{-1} + \right.\right.$$
$$\left.\left. + \frac{3q}{\sqrt{k}} (y_i v_i)^{-1} \right]\right\} > C, \tag{10}$$

where C is the threshold selected in accordance with the accepted criterion, n = 2TΔF, where n is the volume of independent selections of the instantaneous values of the process, T is observation time, and ΔF is the width of the output-process energy spectrum.

If we limit ourselves to the first terms of the expansion ln $l(y)$ (which is admissible for low values of a), expression (10) is rewritten in the following form:

$$\sum_{i=1}^{nb} \left[\left(\frac{3}{2} + \frac{3q}{\sqrt{k}} - \frac{3}{2} q^2 \right) v_i^{-1} - \frac{3q}{\sqrt{k}} (y_i v_i)^{-1} \right] >$$
$$> bn (1 - q)^2 + \frac{1}{a} C = C_1. \tag{11}$$

where b is the ratio of the interval of the nondependence of scattering durations to the magnitude $\frac{1}{2\Delta F}$.

Let us designate the expression under the sum sign in formula (11) by $f(y_1)$. Then the optimum procedure of the treatment is written

$$\sum_{i=1}^{nb} f(y_i) > C_1. \tag{12}$$

If the access volume is large ($bn \gg 1$), the distribution of the sum of independent random magnitudes $\sum_{i=1}^{n} f(y_i)$ is close to the normal value with the average

$$bnm_1\{\eta\} \tag{13}$$

if there is no useful signal, and with the average value

$$bnm_1\{\eta_a\} \tag{14}$$

when there is a useful signal, where

$$m_1\{\eta\} = \int_0^\infty f(y)\, W_1(y)\, dy; \tag{15}$$

$$m_1\{\eta_a\} = \int_0^\infty f(y)\, W_1(y/a)\, dy = m_1\{\eta\} - a(1-q^2)\, m_1\{\eta\} + a m_1\{\eta^2\}, \tag{16}$$

and with dispersion if there is no useful signal

$$bn\left(m_1\{\eta^2\} - m_1^2\{\eta\}\right) \tag{17}$$

Since $a \ll 1$, dispersion (17) as assumed to be identical for the cases of the absence and presence of a signal.

Using formulas (13), (14), and (17), we can write the probability of a false alarm $p_{\text{л.т}}$ and correct observation $p_{\text{п.о}}$:

$$p_{\text{л.т}} = 1 - F\left(\frac{C_1 - bnm_1\{\eta\}}{\sqrt{nbM_2\{\eta\}}}\right); \tag{18}$$

$$p_{\text{п.о}} = 1 - F\left(\frac{C_1 - bnm_1\{\eta_a\}}{\sqrt{nbM_2\{\eta\}}}\right). \tag{19}$$

where $F(z) = \frac{1}{2\pi}\int_{-\infty}^{z} e^{-\frac{t^2}{2}}\, dt.$

From equations (18) and (19) we can find $n_{\text{п.о}}$ (the access volume necessary to observe a signal with probability $p_{\text{п.о}}$ with a given value of $p_{\text{л.т}}$ and $\frac{p_c}{p_ш} = a$):

$$n_{\text{п.о}} = \frac{1}{b}\, \frac{m_1\{\eta^2\} - m_1^2\{\eta\}}{[m_1\{\eta^2\} - (1-q^2)\, m_1\{\eta\}]^2} \times \frac{1}{a^2}\left[\arg F(1 - p_{\text{л.т}}) - \arg F(1 - p_{\text{п.о}})\right]^2. \tag{20}$$

where arg F(z) = t, if F(t) = z.

For the optimum amplitude (square) detector the access volume is written in the following form [6]:

$$n_{a.o} = \frac{2}{a^2}\left[\arg F(1-p_{n.\tau}) - \arg F(1-p_{n.o})\right]^2. \qquad (21)$$

The relative asymptotic effectiveness [3] of the interval optimum detector relative to the amplitude optimum is written as:

$$l_{n.o.a.o} = \lim \frac{n_{a.o}}{n_{n.o}} = \frac{2b\left[m_1\{\eta^2\} - (1-\varepsilon^2)\, m_1\{\eta\}\right]^2}{m_1\{\eta^2\} - m_1^2\{\eta\}}. \qquad (22)$$

The results of calculating the standardized relative asymptotic effectiveness $\frac{l_{n.o.a.o}}{b}$, $m_1\{\eta\}$ and $m_1\{\eta^2\}$ for various values of η and k = 0.01 (for 0.01 and 0.001 the results practically coincide) are shown in the table.

The graph shows the dependence of the standardized relative asymptotic effectiveness on detuning the signal frequency relative to the central frequency of the noise energy spectrum.

The dimensionless quantity σ is plotted along the horizontal axis from −1.8 to +1.8, while the dimensionless quantity $l_{n.o.a.o}$ is plotted on the vertical axis from 0 to 11. The figures 3, 6, and 9 are to valves of the corresponding points of the vertical axis.

From the above graph it can be seen that with an increase in the absolute detuning value, the noiseproof quality of the interval optimum detector increases. To quantitatively evaluate the noiseproof quality of the interval detector it is necessary to know the value of b.

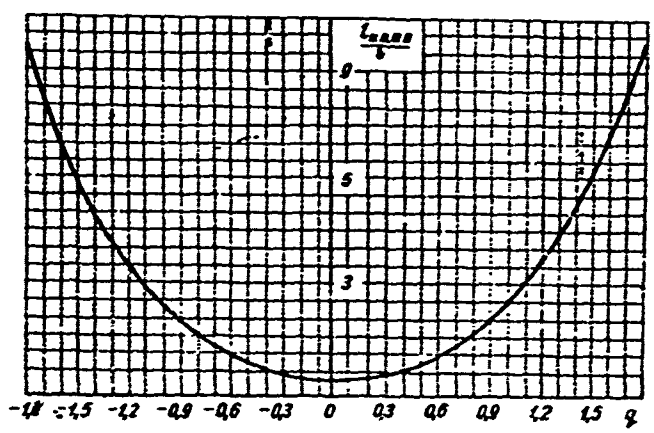
To calculate b it is necessary to find the correlation function of the intervals between zeroes.

$\frac{l_{и.о.а.о}}{b}$	$m_1\{\tau^2\}$	$m_1\{\tau\}$	q
0,397	1,1999	1,00	0,0
0,415	1,1878	0,99	±0,1
0,464	1,1536	0,96	±0,2
0,546	1,1014	0,91	±0,3
0,676	1,0384	0,84	±0,4
0,826	0,974	0,75	±0,5
1,03	0,928	0,67	±0,6
1,31	0,899	0,545	±0,7
1,59	0,922	0,40	±0,8
1,965	1,0146	0,235	±0,9
2,009	1,199	0,05	±1,0
2,93	1,5039	−0,155	±1,1
3,54	1,959	−0,38	±1,2
4,26	2,597	−0,625	±1,3
4,89	3,456	−0,89	±1,4
6,05	4,512	−1,175	±1,5
7,14	5,989	−1,48	±1,6
8,37	7,757	−1,805	±1,7
9,8	9,905	−2,15	±1,8

If we assume that the interval of independence for intervals between zeroes and the phases of the random processes coincide (b = 2), for the signal coinciding with the edge of the square frequency characteristic curve of the filter (q = ±1.76),

$$l_{и.о.а.о} \approx 4{,}5.$$

Thus, under this assumption the noiseproof quality of the optimum Graph of the dependence of the standardized relative asymptotic effectiveness on detuning the signal frequency.



Interval detector is substantially higher than the noiseproof quality of the amplitude detector.

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